

# A SYSTEMATIC TEST OF THE INDEPENDENCE AXIOM

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# MOTIVATION

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*“Consequently, I viewed the principle of independence as incompatible with the **preference for security in the neighborhood of certainty** shown by every subject... This led me to devise some counter-examples. One of them, formulated in 1952, has become famous as the ‘Allais Paradox.’ Today, it is as widespread as its real meaning is generally misunderstood.” - Maurice Allais (emphasis added)*

# ALLAIS PARADOX

- Allais's original intuition was that Independence will be violated “in the neighborhood of certainty,” favoring certainty, but **not** otherwise
- Dillenberger (2010) introduces Negative Certainty Independence (NCI) which allows for exactly this

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- Places Independence restriction only when a lottery is already preferred to a certain amount
- Places no restriction if  $\delta_x \succ p$
- Allows for Independence violations at certainty but not otherwise

## IN THIS PAPER...

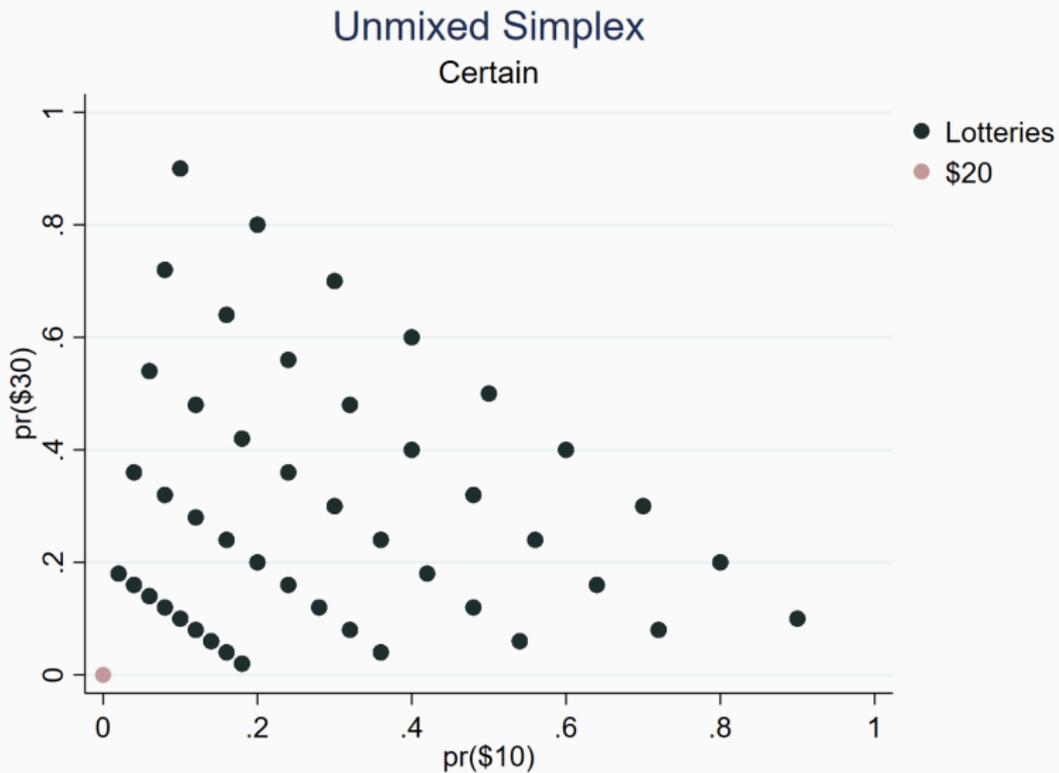
- An extensive literature has found violations of Independence at certainty
- One has found the opposite (Blavatskyy, 2010)
- Few papers have looked away from certainty
- Questions:
  - If we allow for all types of violations, what is most common?
  - Is Independence mostly satisfied away from certainty?
  - Can we explain most violations with a preference for certainty?

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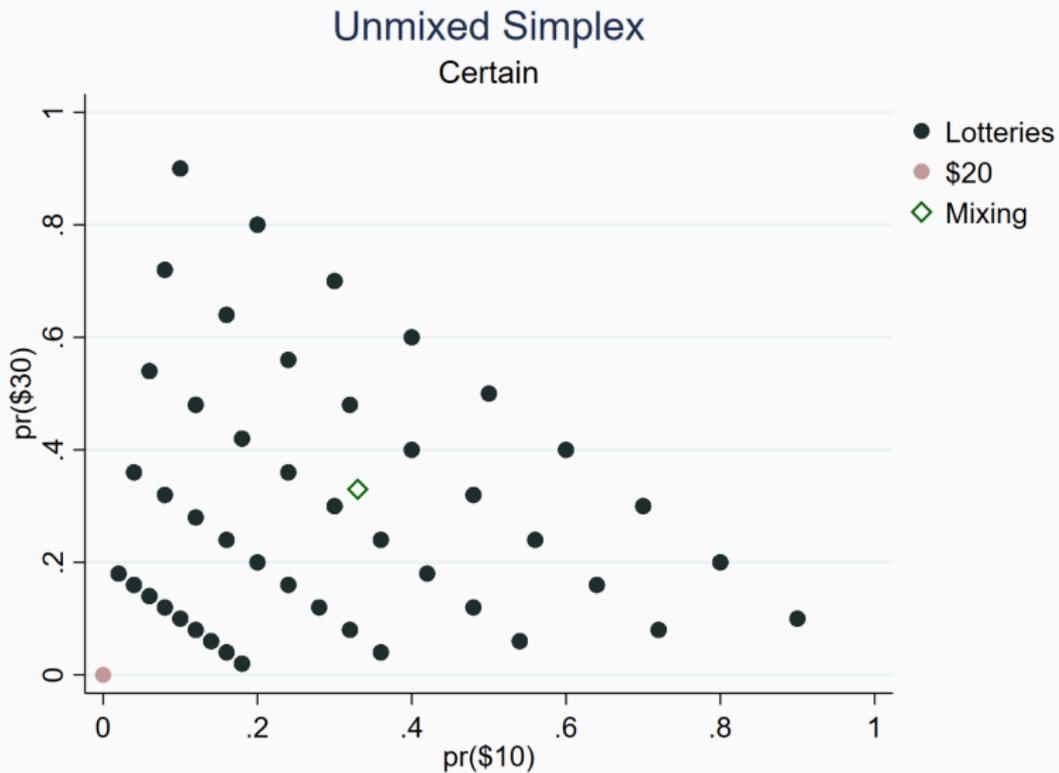
- Test Independence over the entire simplex
- Some decisions involve certainty, some don't
- Look to see where in the simplex Independence is violated most often
  - Whether these involve certainty or not

**DESIGN**

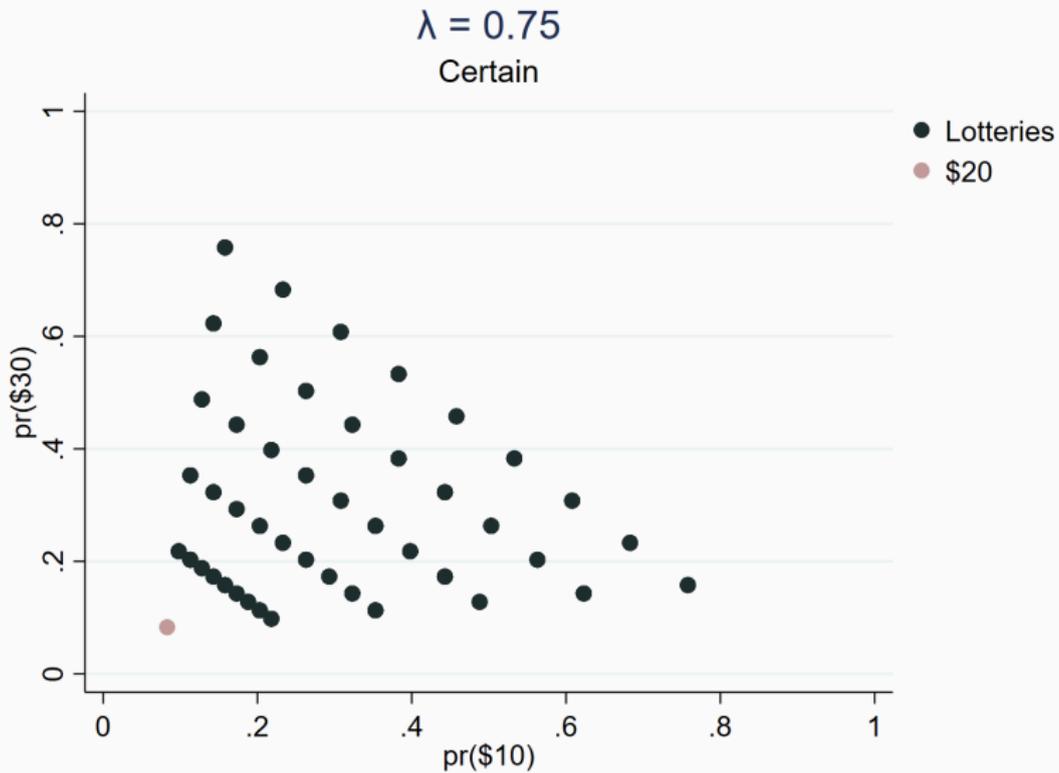
# QUESTIONS



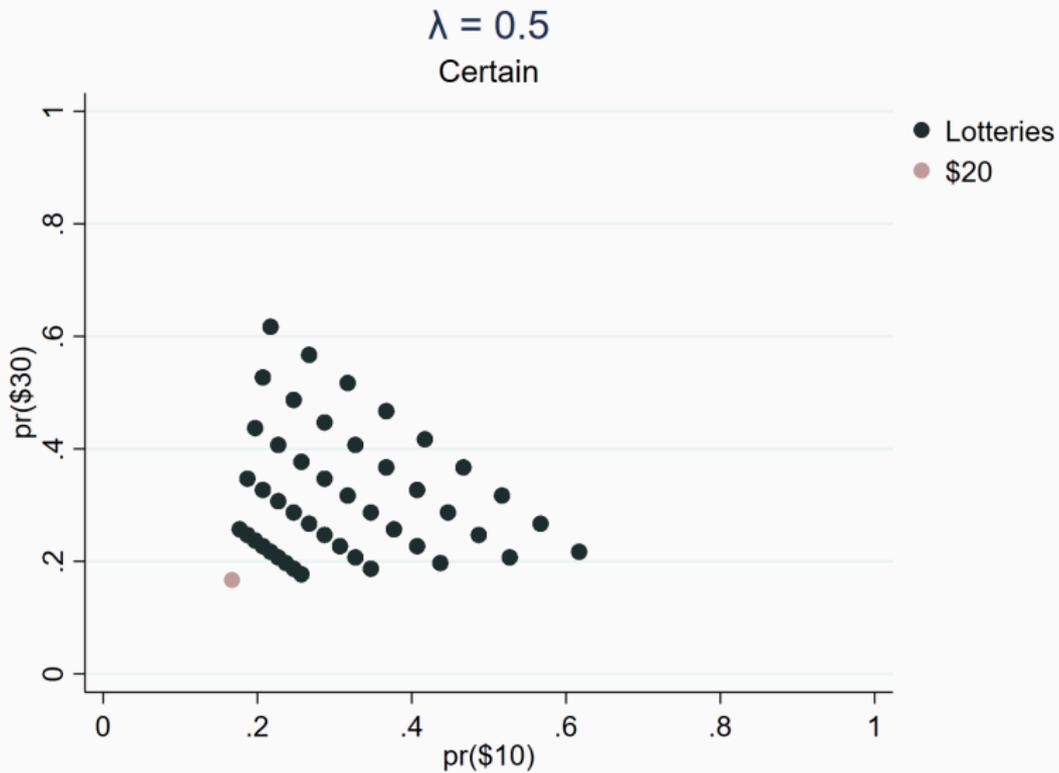
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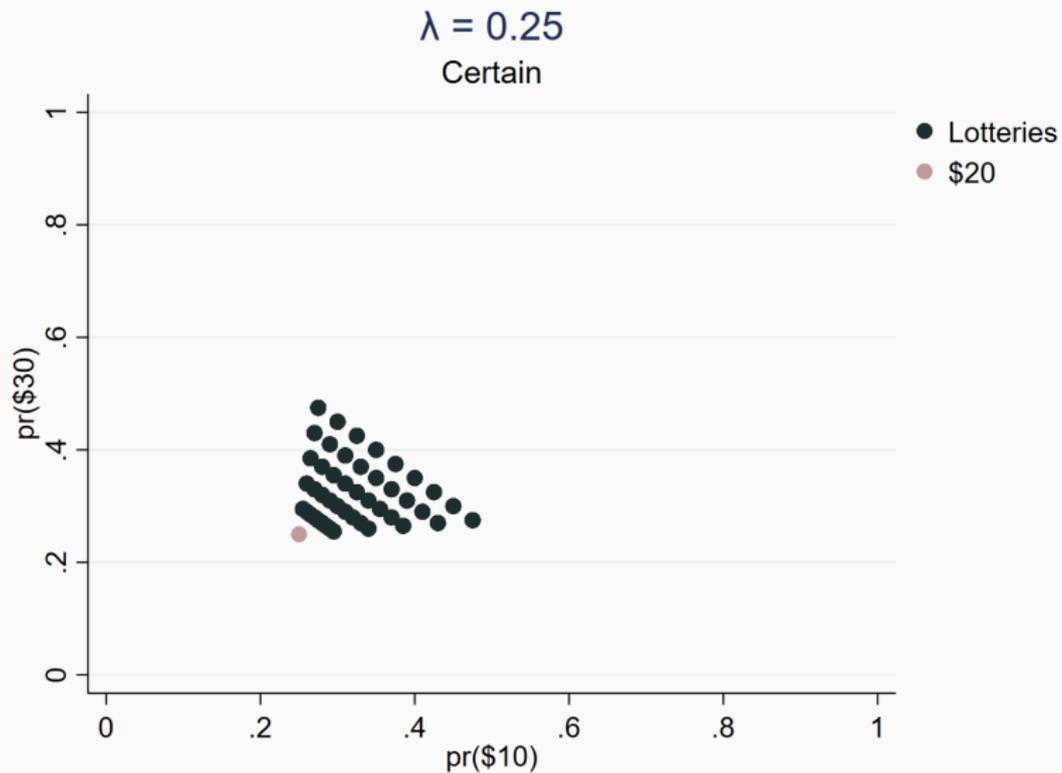
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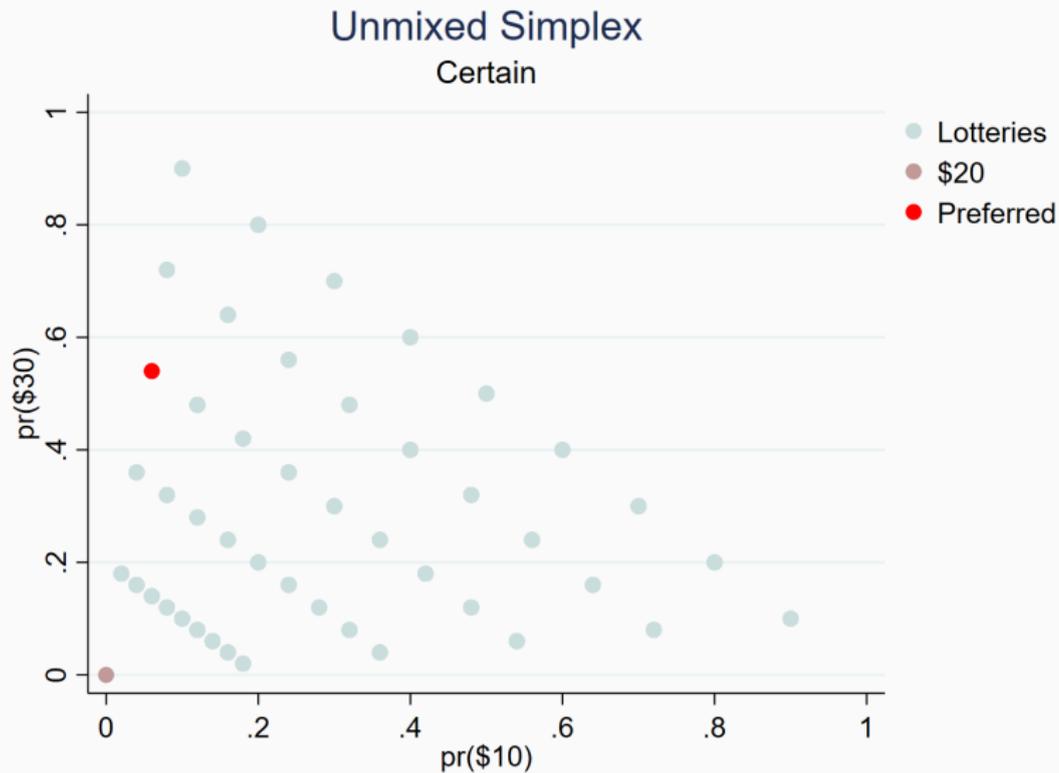
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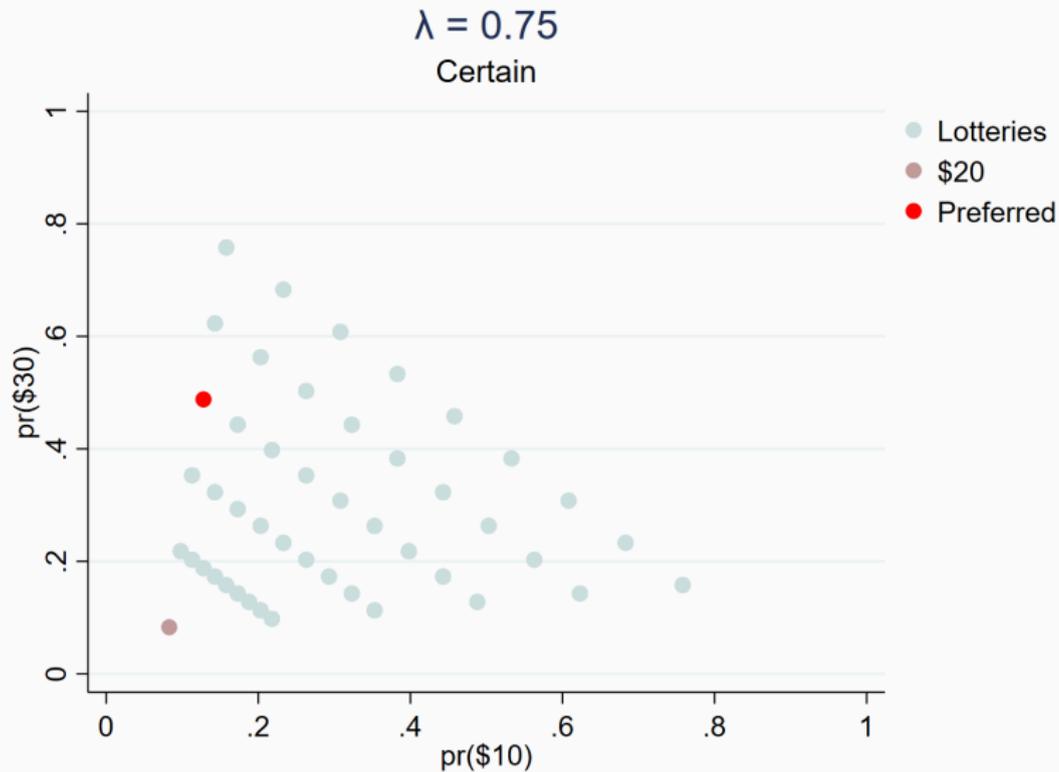
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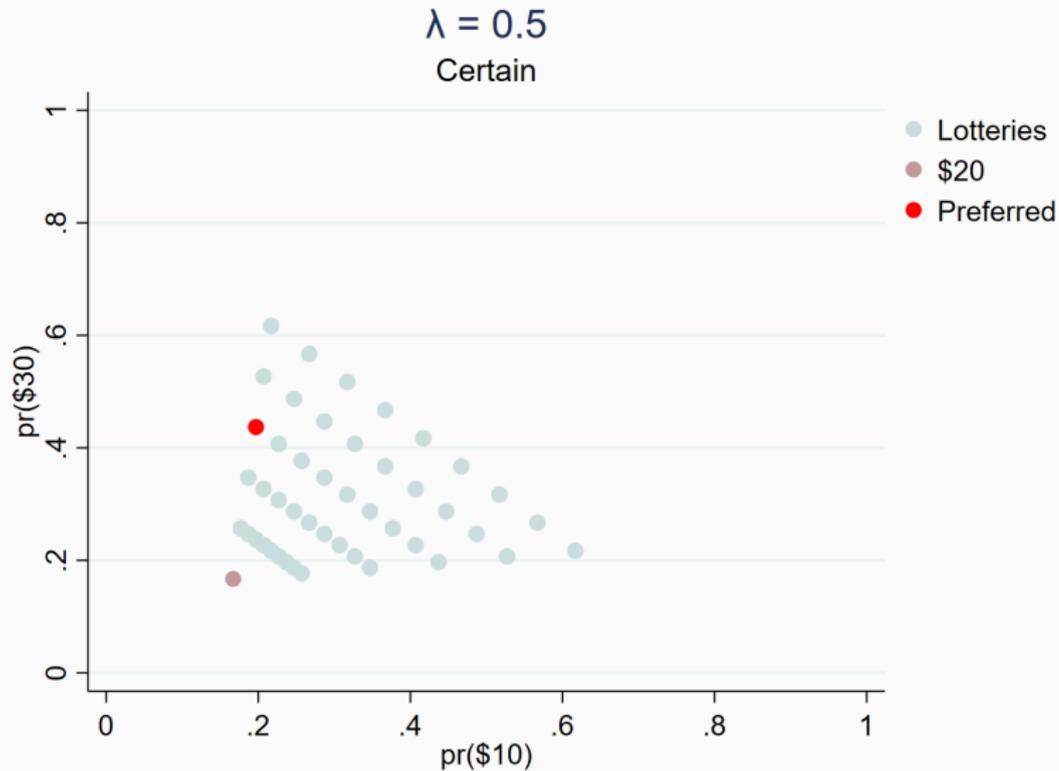
# INDEPENDENCE RESTRICTION



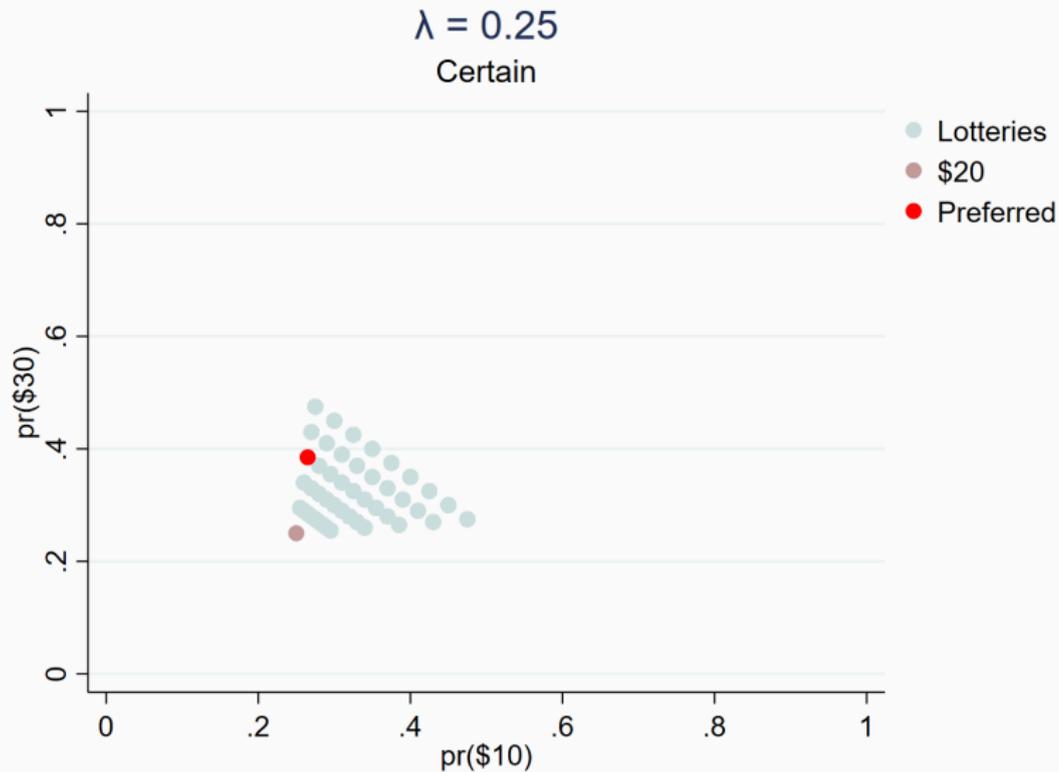
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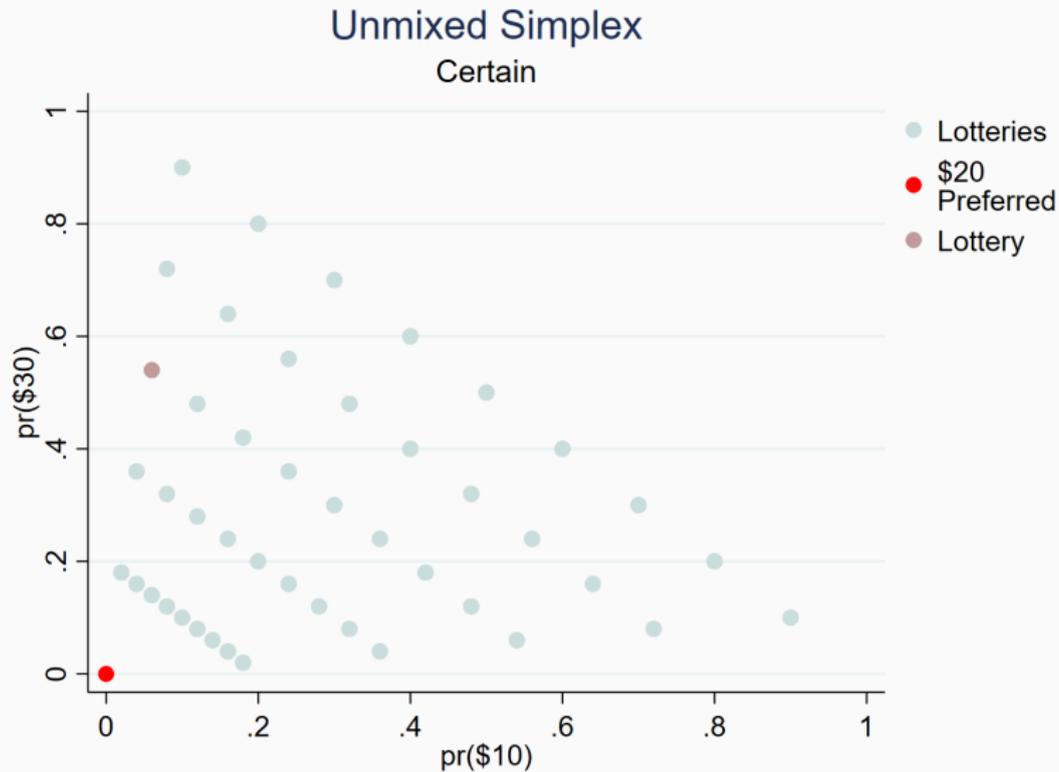
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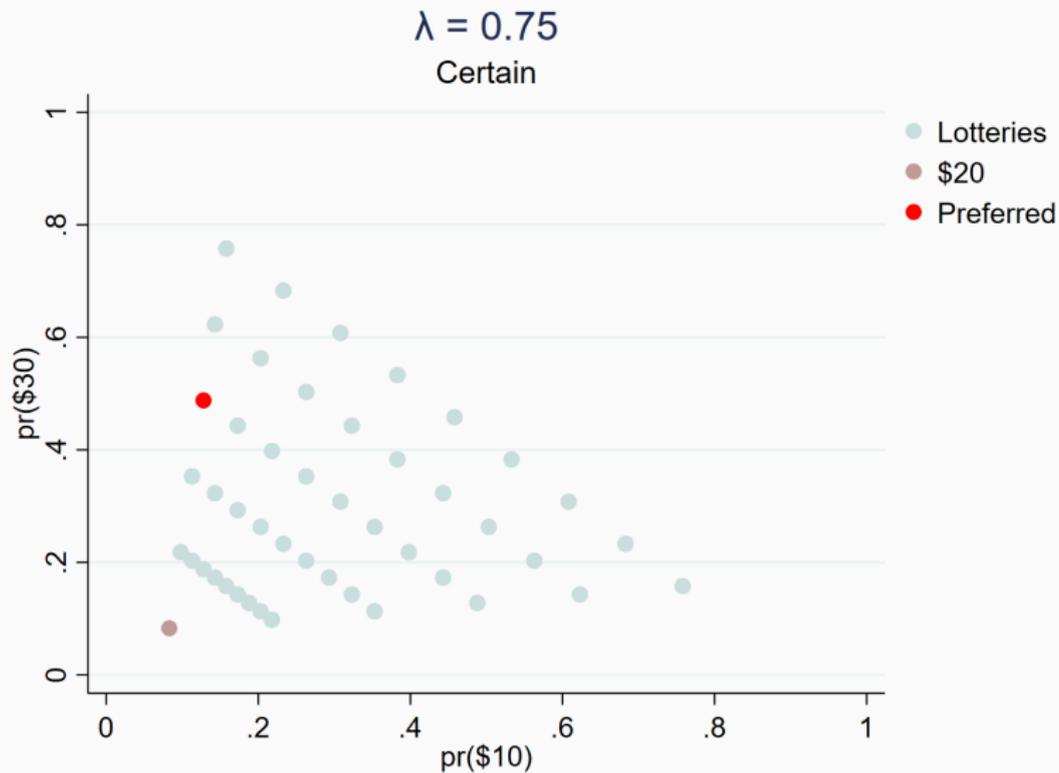
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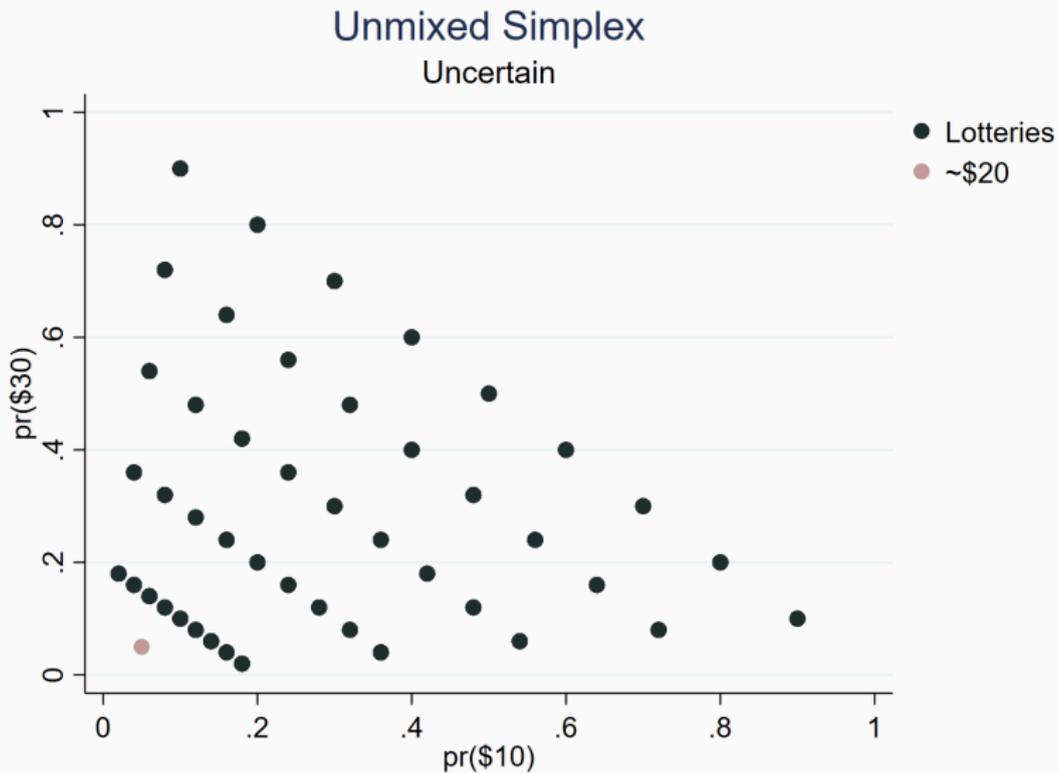
# ALLAIS VIOLATION



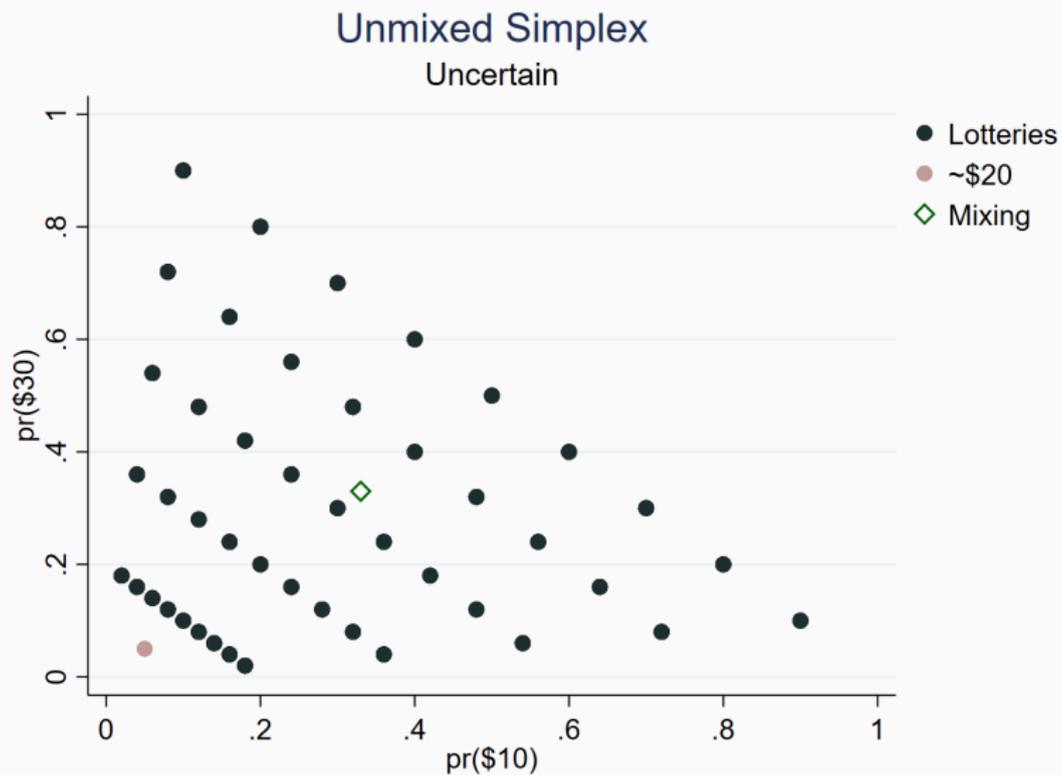
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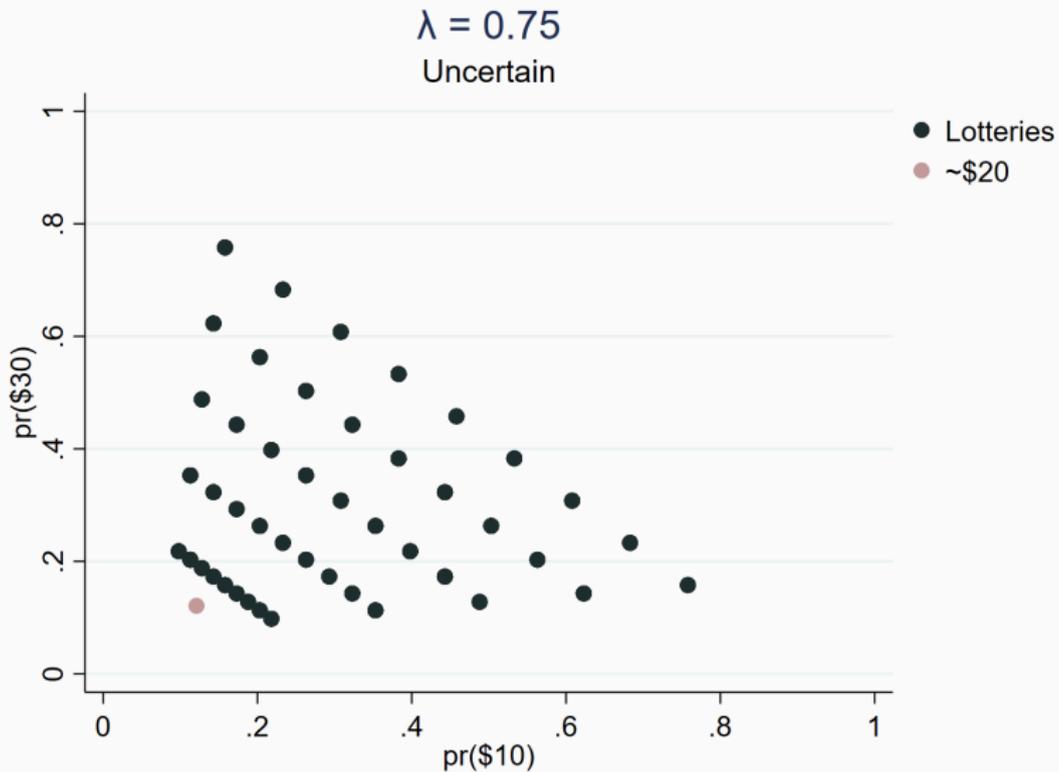
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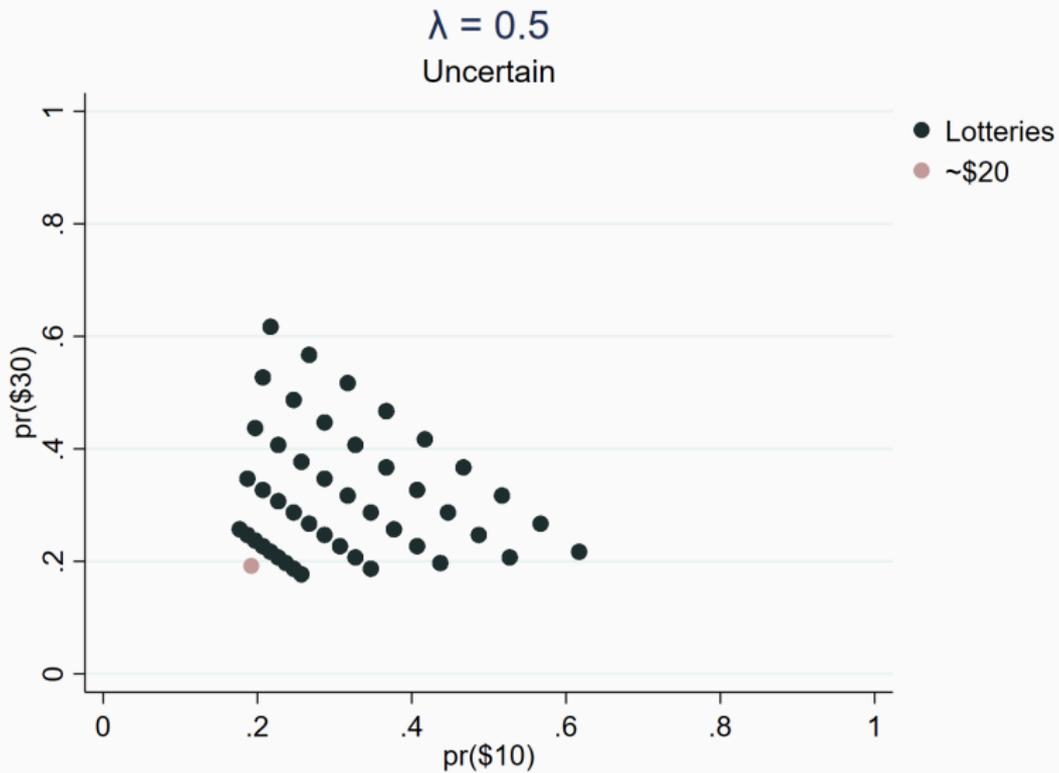
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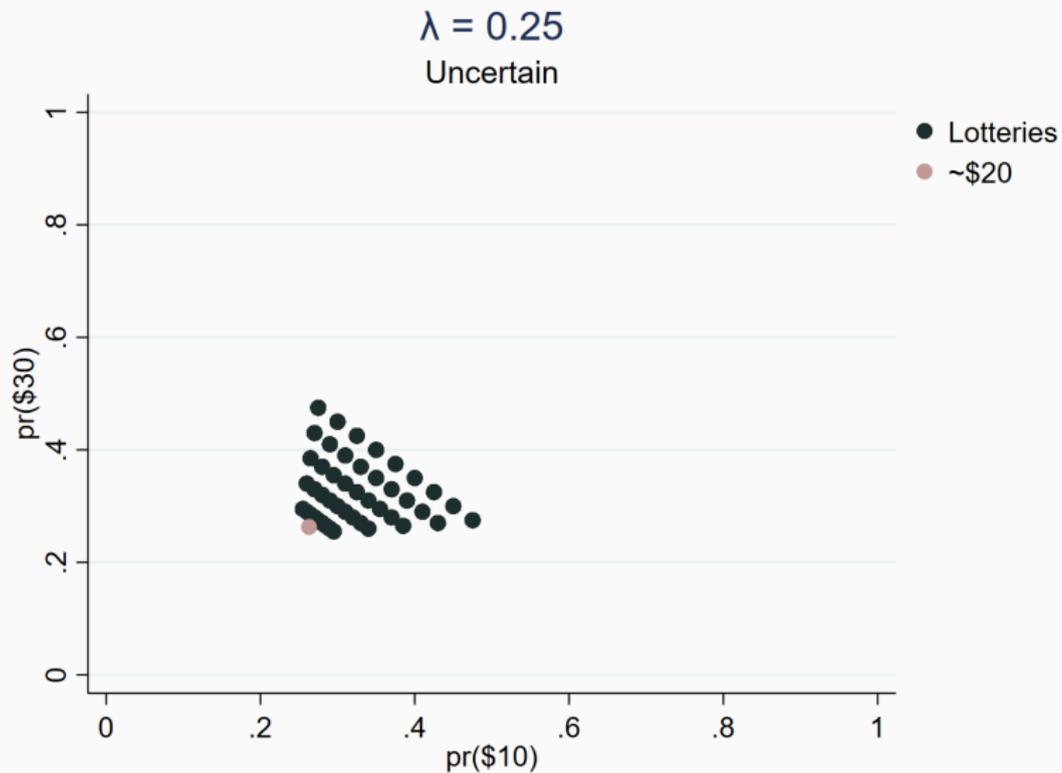
# QUESTIONS



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- 360 total questions
  - (45 questions per simplex)  $\times$  (8 simplices)
- Each subject saw 68 random questions
  - 17 unique  $\lambda = 1$  comparison questions
  - Answered  $\lambda = \{1, 0.75, 0.5, 0.25\}$  for each
  - 51 tests of independence per subject
  - Randomized independently for certainty and uncertainty
- 147 Subjects
- Average payment: \$20/30 min session
  - “High” stakes

ID: 3

Round 5 of 68



\$30: 24.00%  
\$20: 40.00%  
\$10: 36.00%



\$30: 0.00%  
\$20: 100.00%  
\$10: 0.00%

# RESULTS

# RISK PREFERENCE CONSISTENCY



# INDEPENDENCE VIOLATIONS: CERTAIN



# INDEPENDENCE VIOLATIONS: CERTAIN



# INDEPENDENCE VIOLATIONS: UNCERTAIN



# INDEPENDENCE VIOLATIONS: UNCERTAIN

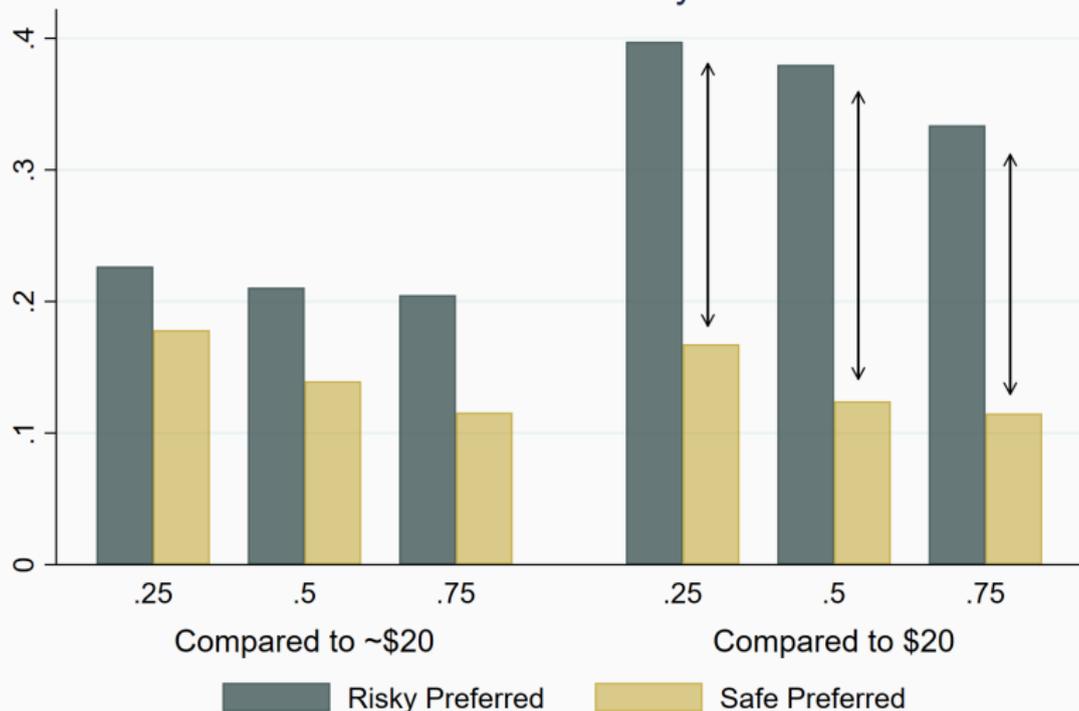


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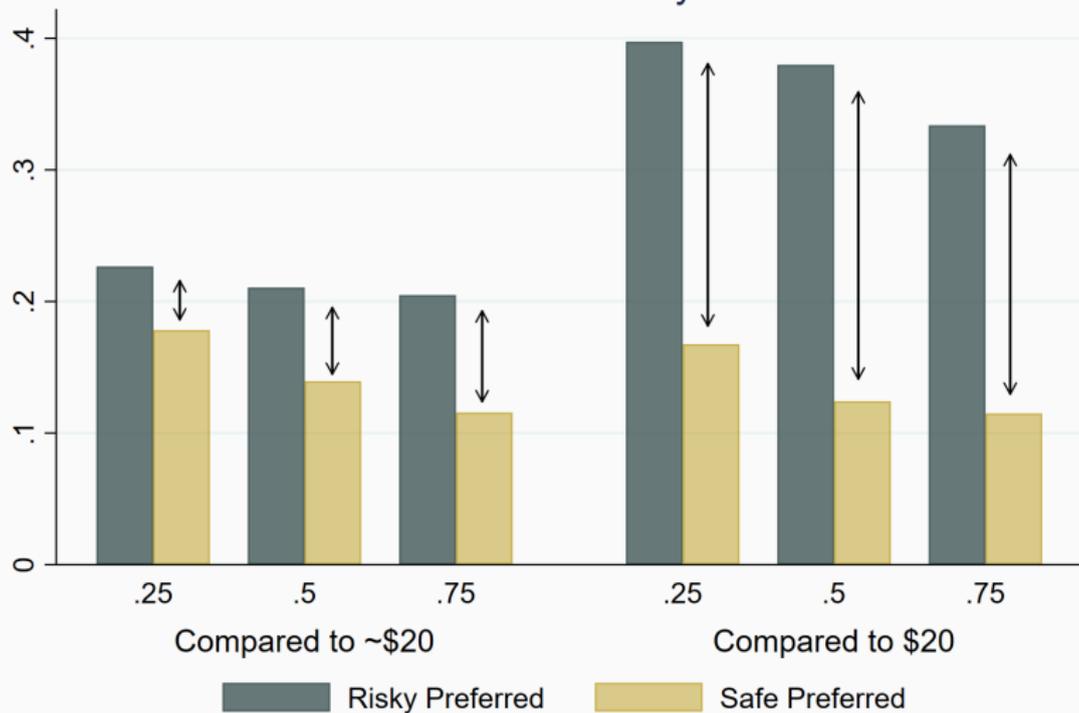
# INDEPENDENCE VIOLATIONS

## REVERSE Certainty Effect



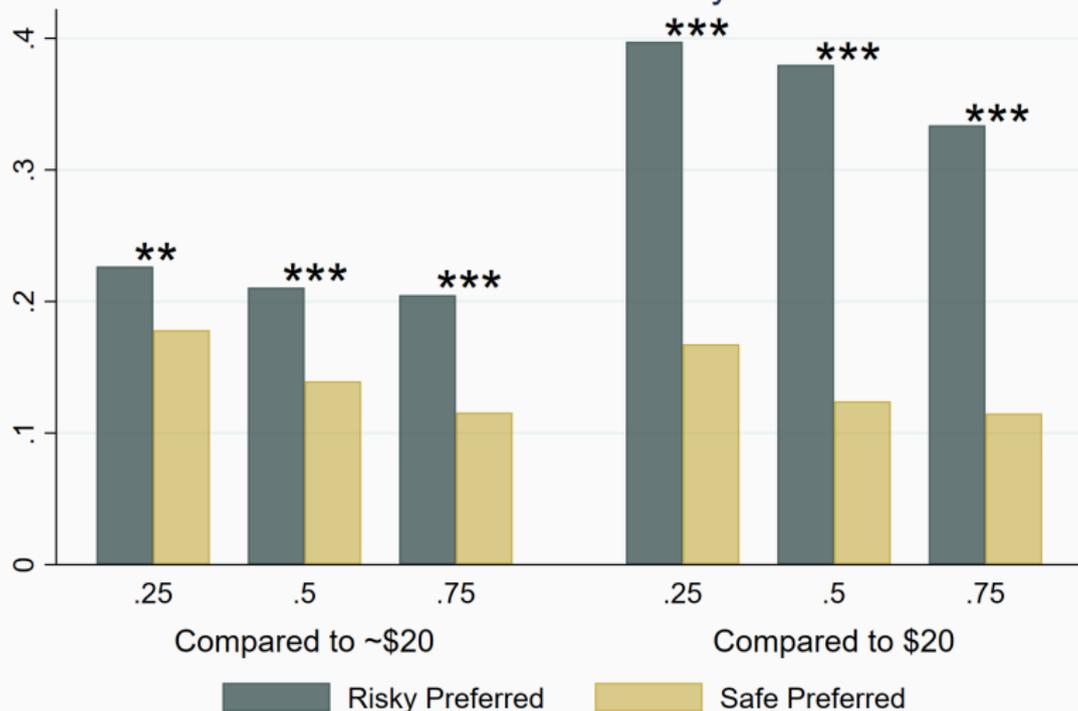
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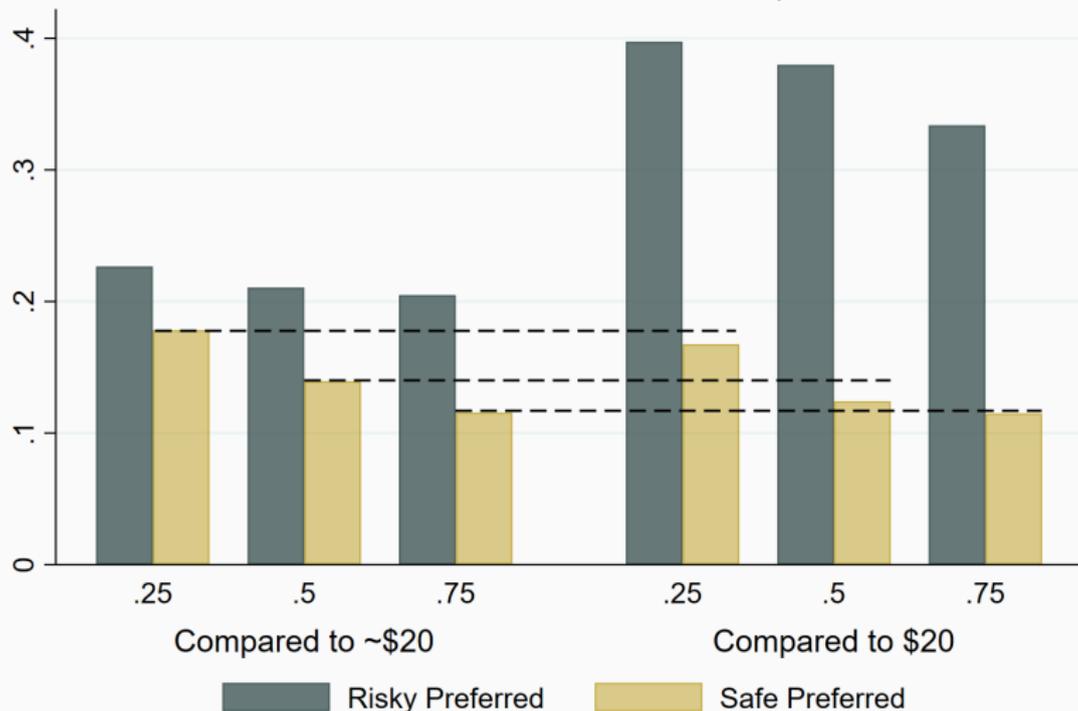
# INDEPENDENCE VIOLATIONS

## More Violations When Risky Preferred



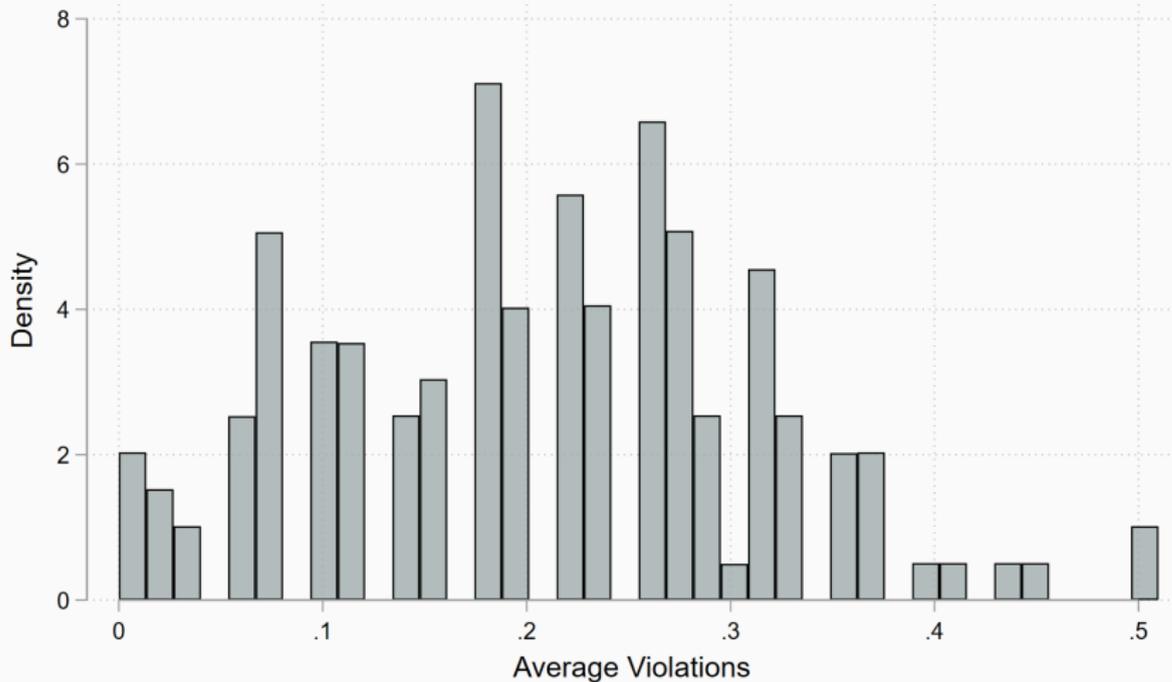
# INDEPENDENCE VIOLATIONS

Safe Preferred Violations Same When \$20 Isn't Certain

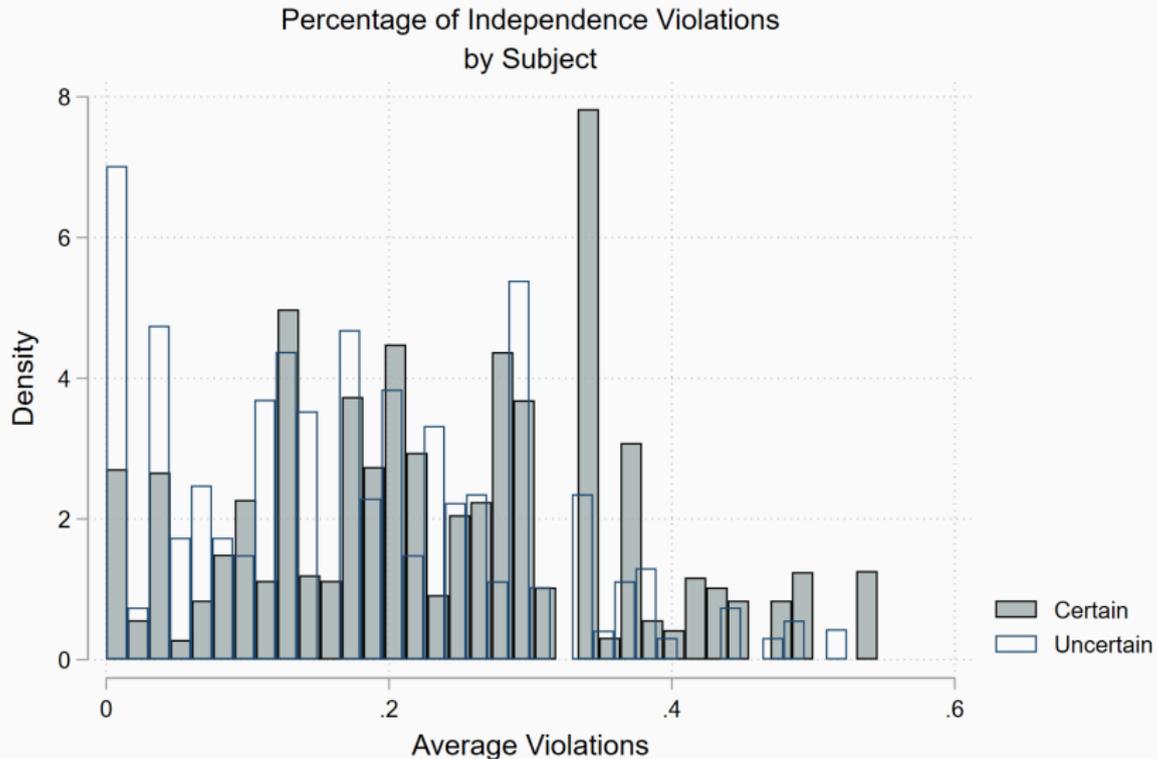


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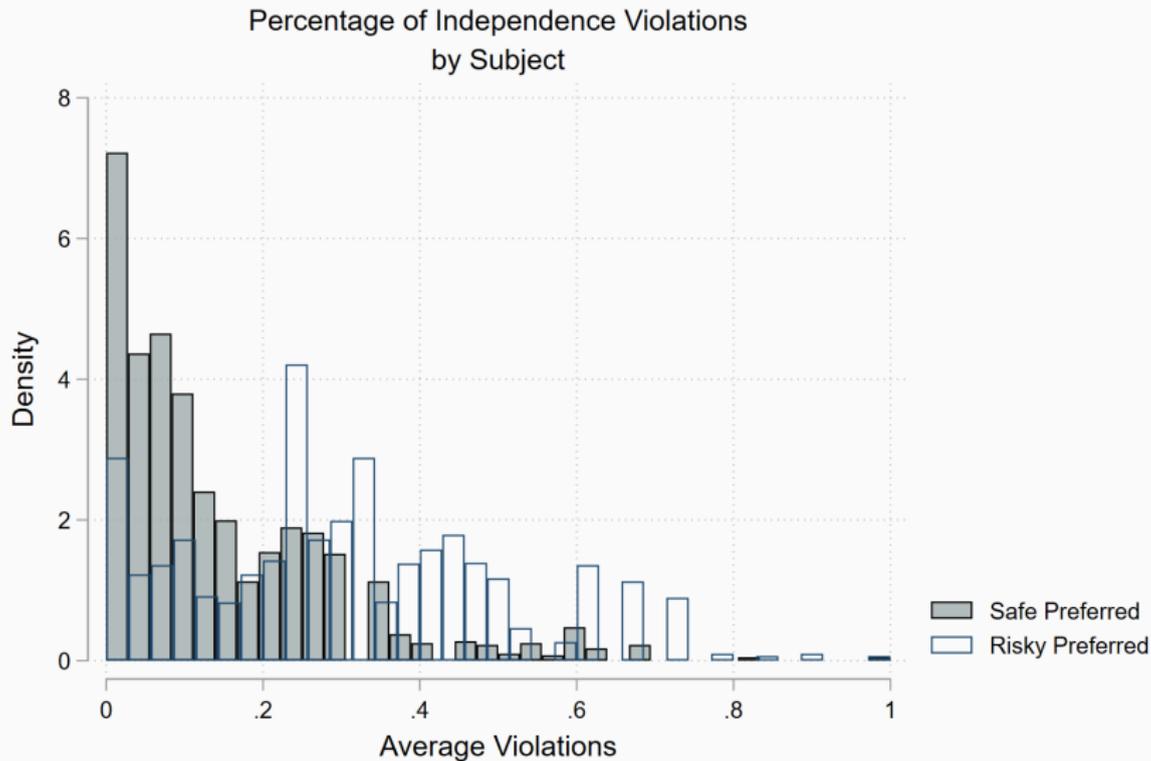
Percentage of Independence Violations  
by Subject



# INDEPENDENCE VIOLATIONS



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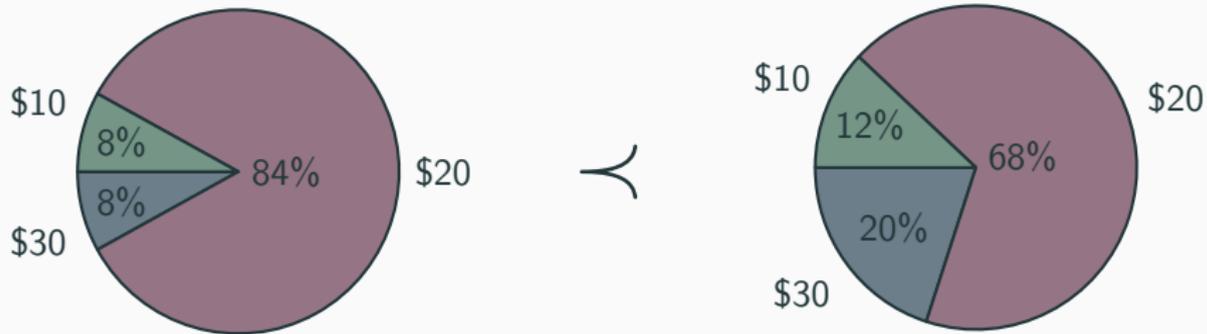
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What does the safe-preferred violation look like?



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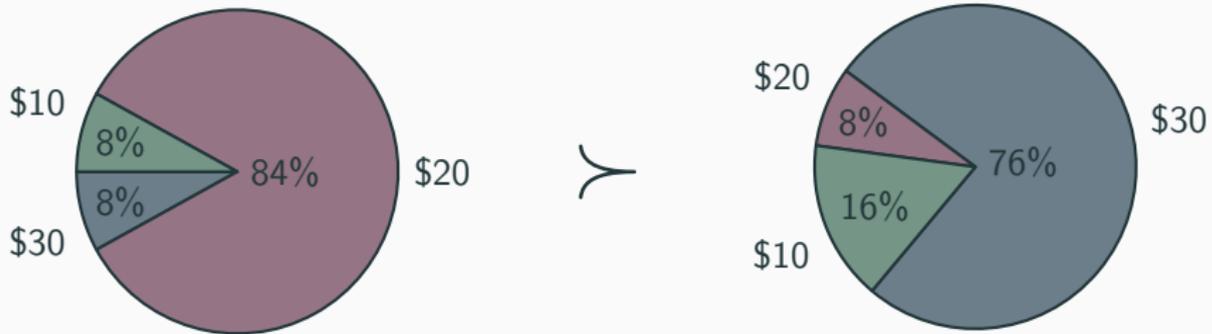
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What does the risky-preferred violation look like?



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What does the risky-preferred violation look like?



# SUMMARY

- We conduct a large-scale test of the independence axiom
- Allow for general violations
- We find the exact opposite of the conventional wisdom
  1. Reverse certainty effect
  2. More violations when the risky lottery is preferred originally
  3. Just as many violations “near” certainty as at certainty itself
- A large literature has studied Allais-type violations
  - In our data, we should be more concerned about the opposite